

Rise time and fall time:

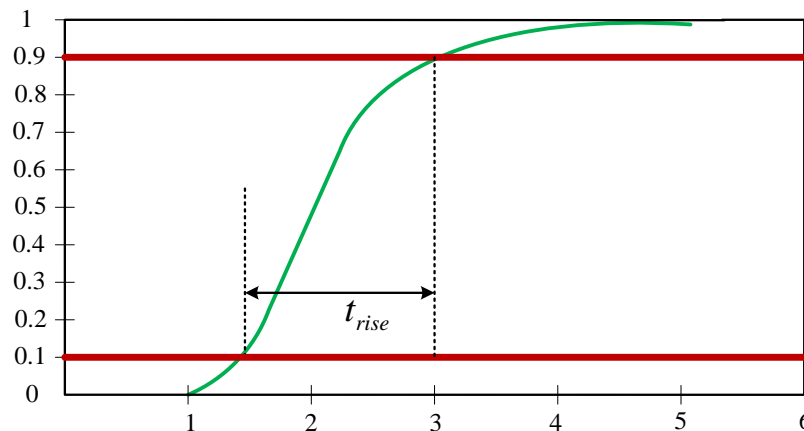
Rise time t_r is defined as the time it takes for a signal to rise from 10% to 90% of its final value.

$$t_r = t_{0.9} - t_{0.1}$$

Where

$t_{0.9}$ = time at which reaches 90% of its steady state value

$t_{0.1}$ = time at which reaches 10% of its steady state value



For a simple one stage low pass RC circuit, rise time is proportional to the circuit to time constant $\tau = RC$.

$$t_r \approx 2.2\tau$$

The proportionality constant can be derived by using the output response of the circuit to step function input signal input signal of V_0 amplitude, aka its step response:

$$V(t) = V_0 \left(1 - e^{-\frac{t}{\tau}}\right) \Leftrightarrow \frac{V(t)}{V_0} = \left(1 - e^{-\frac{t}{\tau}}\right)$$

Solving for 10%

$$\frac{V(t)}{V_0} = 0.1 = \left(1 - e^{-\frac{t_{0.1}}{\tau}}\right) \Rightarrow t_{0.1} = \tau (\ln 10 - \ln 9)$$

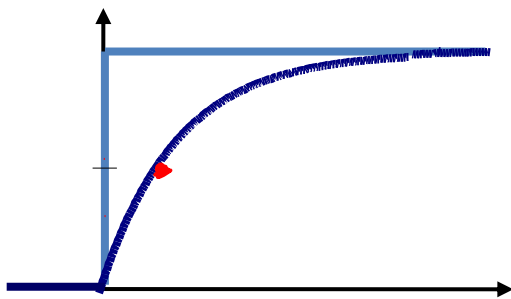
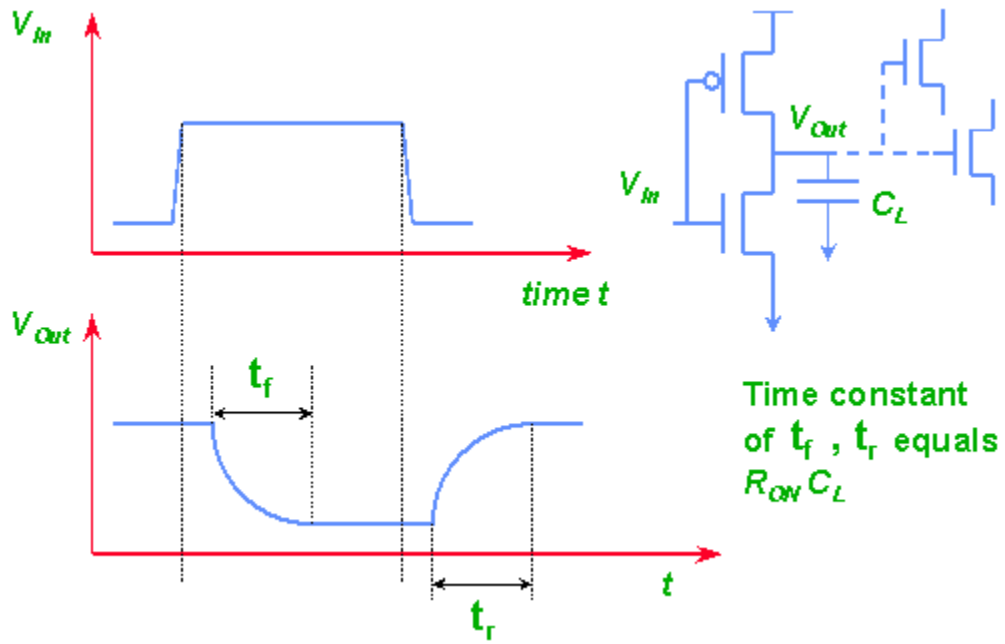
Solving for 90%

$$\begin{aligned} \frac{V(t)}{V_0} = 0.9 &= \left(1 - e^{-\frac{t_{0.9}}{\tau}}\right) \Rightarrow 0.9 - 1 = -e^{-\frac{t_{0.9}}{\tau}} \Rightarrow -0.1 = -e^{-\frac{t_{0.9}}{\tau}} \\ &\Rightarrow \ln(1/10) = \ln(1) - \ln(10) = -\frac{t_{0.9}}{\tau} \Rightarrow t_{0.9} = \tau (\ln 10) \end{aligned}$$

which is the rise time. Therefore rise time is proportional to the time constant:

$$t_r = t_{0.9} - t_{0.1} = \tau (\ln 10) - \tau (\ln 10 - \ln 9) = \tau \ln 9 = 2.197\tau \approx 2.2\tau$$

Real application:



$$V_{out}(t) = V_{supply} (1 - e^{-t/R_p C})$$