Understanding Odds Ratios

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The odds ratio and the risk ratio are related measures of relative risk. The risk ratio is easier to understand and interpret, but, for mathematical reasons, the odds ratio appears more frequently in the medical literature. When the outcome is rare, odds ratios and risk ratios have similar values and can be used interchangeably. However, when the outcome is common, odds ratios should not be interpreted as risk ratios because doing so can greatly exaggerate the size of an effect. This column explains what odds ratios are, how to correctly interpret them, and how to avoid being misled by them.

RISK VERSUS ODDS

A risk is just the probability of an event happening (in a defined time period). An odds is the risk of an event happening divided by the risk of it not happening. Odds are frequently used in gambling, for example, if a sports team is believed to have a 1 in 5 probability of winning (.20), then the odds are 1 to 4 (.25), or 1 win for every 4 losses. More examples are given in Table 1. When viewed as a fraction, an odds is always bigger than its corresponding risk (eg, $1/999 > 1/1000$). Small risks and odds are close in value (eg, 1/1000 versus 1/999), but large risks and odds can be quite different (eg, 9/10 versus 9/1).

RISK RATIOS VERSUS ODDS RATIOS

When a study has a binary outcome (eg, disease or no disease), then investigators can calculate risk ratios and odds ratios. Risk and odds ratios are calculated by using either cumulative risk (from longitudinal studies) or prevalence (from cross-sectional studies). The risk ratio gives the relative increase or decrease in the risk (or prevalence) of the outcome given a particular exposure or treatment; the odds ratio is similar but gives the relative increase or decrease in the odds.

The risk ratio divides the risk (or prevalence) in an exposed group by the risk in a reference group. For example, in a longitudinal study, if 50% of heavy drinkers develop hypertension, then the cumulative risk of hypertension in this group is estimated as 50%. (Or, in a cross-sectional study, if 50% of heavy drinkers have hypertension already, then the prevalence of hypertension in this group is estimated as 50%.) If the risk of hypertension in nondrinkers from the same study is 25%, then the risk ratio is $RR = 50\%/25\% = 2.0$; interpretation: drinkers have twice the risk of hypertension as nondrinkers (a 100% increase in risk). Alternatively, we could calculate the risk ratio by comparing nondrinkers to drinkers: $RR = 25\%/50\% = 0.5$; that is, nondrinkers have half the risk of hypertension as drinkers (a 50% decrease in risk).

The odds ratio divides the odds in the exposed group by the odds in the reference group. For our hypothetical example, the odds of hypertension for drinkers is 50%/50% or 1 to 1; and the odds for nondrinkers is 25%/75% or 1 to 3; thus, the odds ratio is $OR = (50\%/50\%)/(25\%/75\%) = 3.0$, or, equivalently, $OR = (1/1)/(1/3) = 3.0$. This means that drinkers have 3 times the odds of nondrinkers (a 200% increase in odds). Alternatively, the odds ratio that compares nondrinkers with drinkers is: $OR = (1/3)/(1/1) = 0.33$; that is, nondrinkers have one-third the odds of drinkers (a 66.7% decrease in odds).

Note that, if these odds ratios were misinterpreted as risk ratios, then they would overestimate the size of the effect, as we have already seen, risk is doubled, not tripled in drinkers (or cut by half, not by two-thirds for nondrinkers). In this hypothetical example, the distortion is clear

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Table 1. *Examples of risks (given as fractions or percentages) and their corresponding odds (given as fractions)*

Risk	Corresponding Odds
1/1000 (.1%)	1/999
1/100 (1%)	1/99
1/50 (2%)	1/49
1/10 (10%)	1/9
1/4 (25%)	1/3
1/2 (50%)	1/1
9/10 (90%)	9/1
99/100 (99%)	99/1

because the risk ratio is available; but often, only the odds ratio is available (for reasons explained below).

WHY ODDS RATIOS?

Why are odds ratios used at all when risk ratios are easier to understand? The main reason is that **logistic regression**, the multivariate regression technique for modeling binary outcomes, yields odds ratios, not risk ratios. (Odds have better mathematical properties for regression modeling; for example, odds can range from zero to infinity, whereas risks can only range from 0 to 1.) By using logistic regression, investigators can adjust for confounding, examine the effects of multiple predictors simultaneously, and quantify the effects of continuous predictors. Because most investigators want to take advantage of this powerful technique, they wind up reporting odds ratios rather than risk ratios. Odds ratios are also valid in certain situations when risk ratios are not, such as in **case-control studies**.

> **Logistic regression:** models the ln(odds), the natural log of the odds, of the outcome as a function of predictors; estimates adjusted odds ratios for these predictors.

WHEN IS THE ODDS RATIO A GOOD APPROXIMATION OF THE RISK RATIO?

Odds ratios are always a distortion of their corresponding risk ratios. The extent of the distortion depends on the frequency of the outcome under study and the size of the effect (see In-Depth box for mathematical details). When the outcome is rare, the distortion is small; in this case, the odds ratio provides a good approximation of the risk ratio and can be interpreted as such. But, when the outcome is common, the distortion can be large and the odds ratio should not be interpreted as a risk ratio. Larger effect sizes (bigger differences between the groups) also magnify the distortion. These relationships are graphically illustrated in Figure 1. As a general rule of thumb, it is acceptable to interpret the odds ratio as a risk ratio when the risk (or prevalence) of the outcome in the reference group is less than 10% [1,2]. In most cases, the odds ratio and risk ratio are similar when the outcome is this rare (Figure 1).

WHEN CAN ODDS RATIOS MISLEAD?

> **Case-control study:** investigators recruit participants who already have a disease and controls without the disease. Because participants are selected based on their disease status, it is not possible to calculate the risk of disease.

When outcomes are common, the odds ratio should not be interpreted as a risk ratio or this can result in misleading and erroneous statements, as illustrated by the examples that follow (summarized in Table 2).

Vgontzas et al [3] performed a cross-sectional study that looked at the relationship between sleep characteristics and hypertension. After adjusting for potential confounders by using logistic regression, they found strong associations between sleep problems and hypertension. The odds ratios for hypertension in the 2 highest-risk sleep groups (insomniacs who slept ≤5 hours or 5-6 hours per night) compared with the reference group (good sleepers) were 5.12 and 3.53, respectively. The investigators concluded that these groups have a "risk of hypertension 500% or 350% higher" than the reference group; and this interpretation was widely repeated in media coverage of the study. However, it is easy to see that this exaggerates the effect. The reference group had a hypertension prevalence of about 25%; thus; a 5-fold higher risk would put the prevalence of hypertension in the highest-risk group at 125%, a clear impossibility. For cross-sectional and cohort studies, one can convert an odds ratio from logistic regression into an estimate of the risk ratio by

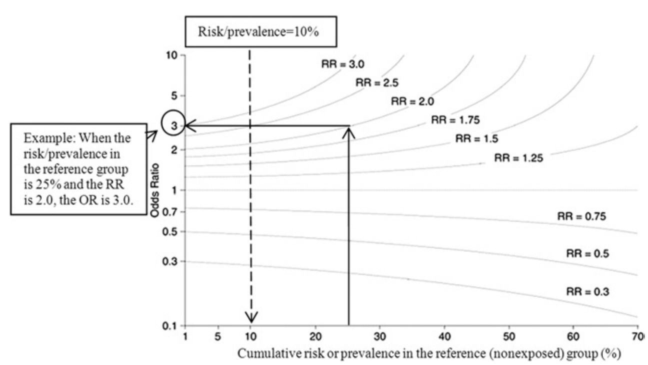

Figure 1. Illustration of the distortion between the odds ratio and the risk ratio (RR). The solid black arrows give one example of how to read the graphic. When the prevalence is 25% and the risk ratio is 2.0 (as in the hypothetical hypertension example), then the odds ratio will be 3.0. The area to the left of the dashed line represents the region in which outcomes are considered rare (less than 10%), the odds ratio and risk ratio are similar in this region for most effect sizes. Reproduced with permission from Reference 1.

IN DEPTH: The mathematical relationship between the odds ratio and the risk ratio

The risk ratio (RR) and odds ratios (OR) can be represented mathematically as:

$$RR=\frac{p_{\exp}}{p_{ref}}$$

p_{exp}= the probability (risk) of the outcome in the "exposed" group
p_{ref} = the probability (risk) of the outcome in the reference group

$$OR=\frac{(\frac{p_{\exp}}{1-p_{\exp}})}{(\frac{p_{ref}}{1-p_{ref}})}$$

1-p_{exp} = the probability (risk) of the outcome NOT occurring in the "exposed" group
1-p_{ref} = the probability (risk) of the outcome NOT occurring in the reference group

Thus, the mathematical relationship between the OR and RR is:

$$OR=RR*\frac{(1-p_{ref})}{(1-p_{\exp})}$$

That is, the OR equals the RR times a "distortion term," $\frac{(1-p_{ref})}{(1-p_{\exp})}$

When the outcome is rare (p_{ref} is small), then the distortion term is close to 1.

Example: if p_{exp}=2% and p_{ref}=1%, then $\frac{(1-p_{ref})}{(1-p_{\exp})}=\frac{99\%}{98\%}\approx 1$

But when the outcome is common, the distortion term is larger.

Example: if p_{exp}=50% and p_{ref}=25%, then $\frac{(1-p_{ref})}{(1-p_{\exp})}=\frac{75\%}{50\%}=1.5$

Bigger differences between the exposed and reference groups (bigger effect sizes), also give larger distortions.

Example: if p_{exp}=50% and p_{ref}=10%, then $\frac{(1-p_{ref})}{(1-p_{\exp})}=\frac{90\%}{50\%}=1.8$

The mathematical relationship between the OR and RR can also be expressed as:

$$RR=\frac{OR}{(1-p_{ref})+(p_{ref}*OR)}$$

using a simple formula [1]. Here, the odds ratios, 5.12 and 3.53, translate into risk ratios of 2.5 and 2.2, respectively. Thus, risk is doubled, not quintupled or tripled [4].

As another example, take a cross-sectional study that looked at the relationship between smoking and wrinkles [5]. After adjusting for potential confounders by using logistic regression, the investigators found a strong relationship between a history of heavy smoking and the presence of prominent facial wrinkling. The odds ratio that compared heavy smokers to nonsmokers was 3.92. The investigators concluded that a heavy smoker "has 3.92 times the risk of developing prominent wrinkles as does a nonsmoker." However, because 45% of the nonsmoker group had prominent wrinkles, this would put the prevalence in the heavy smoker group at a nonsensical 180%. I estimate that the risk ratio is actually about 1.7. So, risk is increased 70%, still a large and important amount but not as dramatic as a 290% increase.

Table 2. *Examples that illustrate the extent to which odds ratios from logistic regression can overestimate risk ratios when the outcome is common*

Example	Odds Ratio From Logistic Regression	Incorrect Interpretation	Estimated Outcome Prevalence (reference group), %	Estimated Risk Ratio*	Correct Interpretation
Sleep and hypertension (3)	5.12	500% higher risk	25	2.5	150% increase in risk (prevalence)
Sleep and hypertension (3)	3.53	350% higher risk	25	2.2	120% increase in risk (prevalence)
Wrinkles and smoking (5)	3.92	3.9 times higher risk	45	1.7	70% increase in risk (prevalence)

*Estimated by using the formula provided in Reference 1.

HOW TO SPOT MISLEADING ODDS RATIOS

A few simple tricks can help readers spot misleading odds ratios. These work only for study designs in which it is possible to directly calculate risk (or prevalence), such as cohort or cross-sectional studies.

Consider Risk Ratio Ceilings

The risk ratio always has a maximum possible value. For example, if the risk in the reference group is 50%, then risk cannot be more than doubled; otherwise, the exposed group will have a risk greater than 100% (see Table 3 for more examples). In the wrinkles study, because the prevalence of wrinkles is about 45% in nonsmokers, the maximum possible risk ratio is 100%/45% = 2.2. Because the odds ratio (3.92) exceeds this value, it likely greatly overestimates the risk ratio. Investigators often fail to provide readers with the risk in the reference group, but it is usually possible to (roughly) estimate this value from other data in the article.

Examine Absolute Risks

Investigators should report the absolute risk of the outcome in the different groups under study. Although not adjusted for confounders, these values will give readers a sense of the magnitude of the effect, both in relative and absolute terms. For example, if the prevalence of hypertension is 20% in reference group and 35% in the exposed group, then the unadjusted risk ratio is 1.75, and the increase in risk is likely to be in this vicinity, even after adjusting for confounding. Furthermore, the absolute risk difference is 15% (35% versus 20%), arguably a more informative number than the relative risk. Unfortunately, investigators often omit this information (it was not given in the articles described above), and it may be difficult to estimate directly.

Table 3. *Maximum possible value of the risk ratio for a given risk (or prevalence) in the reference group (p_{ref}); calculated as* 100%/p_{ref}

Risk/Prevalence in the Reference Group (p_{ref}), %	Maximum Possible Risk Ratio (100%/p_{ref})
10	10.0
20	5.0
30	3.3
40	2.5
50	2.0
60	1.66

Estimate the Adjusted Risk Ratio

The odds ratio and the risk ratio have a direct mathematical relationship (see In-Depth box). By using this relationship, Zhang and Yu [1] derived a simple formula for converting an adjusted odds ratio (from logistic regression) into a crude estimate of the adjusted risk ratio:

$$RR = \frac{OR}{(1 - p_{ref}) + (p_{ref} * OR)}$$

p_{ref} is the risk of the outcome in the reference group. For example, in the sleep and hypertension article described above, I estimated p_{ref} at about 25%. Thus, the odds ratio of 5.12 can be converted into a risk ratio, as follows:

$$RR = \frac{5.12}{(1 - .25) + (.25 * 5.12)} = 2.5$$

Although this formula gives only a crude estimate of the adjusted risk ratio [2], readers can use it to get a sense of how much the odds ratio distorts the risk ratio.

CONCLUSIONS

When a study has a binary outcome, investigators typically use logistic regression to analyze the data. Logistic regression yields odds ratios. If the outcome is rare (occurring in less than 10% of the reference group), then the odds ratio closely approximates the risk ratio and can be interpreted as a relative increase or decrease in risk. However, if the outcome is common, as is the case for many studies in physiatry, then interpreting odds ratios as risk ratios can lead to exaggerated statements about the size of the effect. To avoid being misled, readers may use a simple formula to convert odds ratios into approximate adjusted risk ratios. They should also look for information on absolute risks, which are often more informative than relative risks.

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