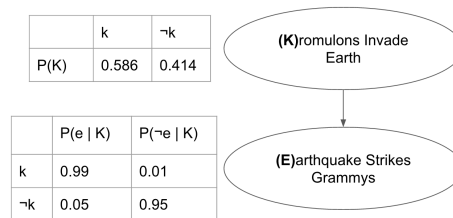# CS188 Fall 2015 Section 6: Bayesian Networks

1 It's Time to Get Schwifty

Welcome to the universe of Rick and Morty, full of dangerous misadventures through the cosmos that often end in dramatic, surreal catastrophe. This time, a giant head has entered Earth's gravity, triggering climate change and natural disasters we thought were impossible for at least another eight years! Luckily, Rick has seen enough of the galaxy to know that what we've got here is a kromulon from the Signes V Expanse. They feed on the talent and showmanship of less-evolved lifeforms. What Earth really needs is a musician to produce a new hit single. But can we get one from the Grammys to the soundstage in Area 51 in time? Let's find out.

	k	¬k
P(K)	0.586	0.414

(K)romulons Invade Earth

	P(e | K)	P(¬e | K)
k	0.99	0.01
¬k	0.05	0.95

(E)arthquake Strikes Grammys

1. Fill in the joint probability table over K and E.

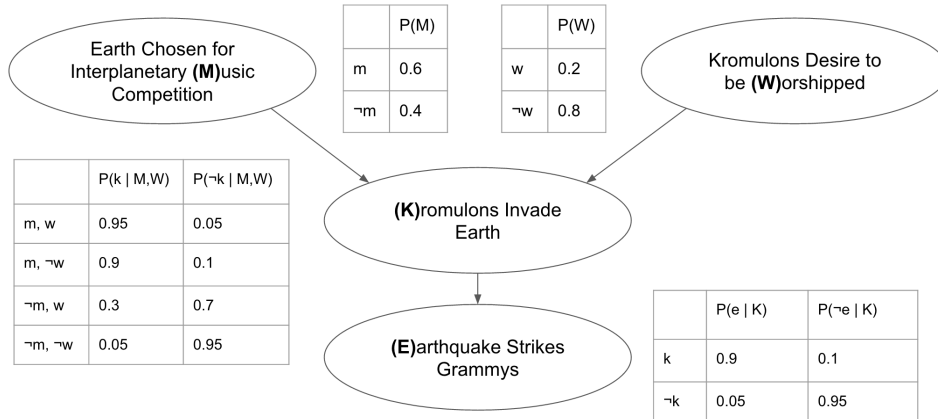| K	E	P(K, E)
k	e	0.58014
k	¬ e	0.00586
¬ k	e	0.02070
¬ k	¬ e	0.39330

2. The high values along the diagonal of $P(E \mid K)$ indicate that these two events are strongly correlated. What is P(e), the marginal probability that an earthquake strikes at the Grammys?

   $P(e) = P(e, k) + P(e, \neg k) = P(e \mid k)P(k) + P(e \mid \neg k)P(\neg k) = (.99)(.586) + (.05)(.414) = .60084$

3. Ice-T was at the Grammys, and managed to survive the earthquake (e). He hasn't seen the kromulon yet, so what does he think is the probability $P(k \mid e)$ that earth was invaded?

   $$P(k \mid e) = \frac{P(k, e)}{P(e)} = \frac{P(e \mid k)P(k)}{P(e)} = \frac{(.99)(.586)}{.60084} \approx .966$$

   Rick, Morty, and the president rejoice after successfully impressing the kromulon with their hit song. However, Earth is then immediately teleported to another universe full of kromulons! Let's speculate about the kromulons' true intentions. Rick believes that Earth has been entered into a musical reality show, whereas everyone in their hometown seems convinced that the many giant heads want to be worshipped as gods. Let's expand our model to consider these potential causes.

	P(M)
m	0.6
¬m	0.4

	P(W)
w	0.2
¬w	0.8

	P(k | M,W)	P(¬k | M,W)
m, w	0.95	0.05
m, ¬w	0.9	0.1
¬m, w	0.3	0.7
¬m, ¬w	0.05	0.95

	P(e | K)	P(¬e | K)
k	0.9	0.1
¬k	0.05	0.95

4. The full joint distribution is given below. Fill in the missing values. Take + and - to be shorthand for true and false.

M	W	K	E	$P(M,W,K,E)$	M	W	K	E	$P(M,W,K,E)$
+	+	+	+	0.1026	-	+	+	+	0.0216
+	+	+	-	0.0114	-	+	+	-	0.0024
+	+	-	+	0.0003	-	+	-	+	0.0028
+	+	-	-	0.0057	-	+	-	-	0.0532
+	-	+	+	0.3888	-	-	+	+	0.0144
+	-	+	-	0.0432	-	-	+	-	0.0016
+	-	-	+	.0024	-	-	-	+	0.0152
+	-	-	-	.0456	-	-	-	-	0.2888

1. Compute the following quantities. You may use either the full joint distribution or the conditional tables, whichever is more convenient.

2. P(m, k) = $\sum_{W,E} P(m,W,k,E)$ = .1026 + .0114 + .3888 + .0432 = $\boxed{.546}$ (calculated by marginalizing twice from joint)

3. P(e, k | m) = $P(e|k)P(k|m) = P(e|k)[P(k|m,w)P(w) + P(k|m,\neg w)P(\neg w)] = (.9)[(.95)(.2) + (.9)(.8)]$ = $(.9)(.91) = \boxed{.819}$ (used $E \perp\!\!\!\perp M|K$, and found P(k|m) by summing out W)

4. P(e | ¬ w,k) = $P(e|k) = \boxed{.9}$ (directly from table, since $W \perp\!\!\!\perp E|K$)

5. P(m | k) = $\frac{P(m,k)}{P(k)} = \frac{.546}{.586} \approx .932$

(Used calculations from part 2, and summed from joint. Note that this directly indicates that the music show is quite likely, given the invasion. The same calculation for the worshipping gives a much smaller number.)

1. It turns out there is another alien species, the Normoluks from the Signes VI Expanse, that holds similar music reality shows. If Earth was chosen to participate, they would invade, and cause natural disasters in much the same way. Add a node N to the Bayes net that reflects this possibility. Draw the new Bayes net below. Which CPT or CPT's need to be modified?
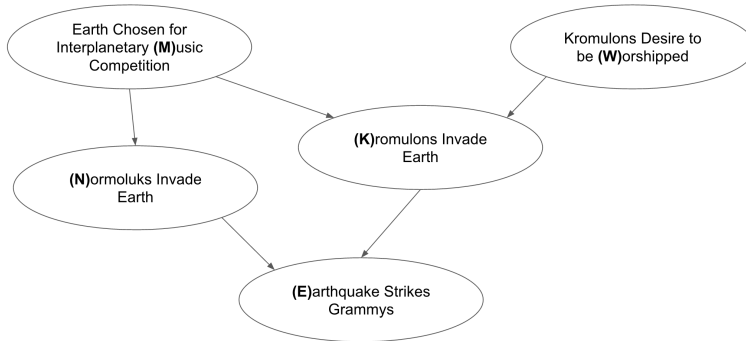
We will add a table $P(N|M)$, for the new node. The parent node, M, will remain unaffected. $P(E|K)$ will become $P(E|N,K)$, and will contain 8 entries instead of 4.

2. Consider your augmented model. Just based on the structure, which of the following are guaranteed to be true? The image on the right may help.

1. $M \perp\!\!\!\perp E|K$ Not indicated by structure

2. $M \perp\!\!\!\perp E|K, N$ True

3. $M \perp\!\!\!\perp W$ True

4. $M \perp\!\!\!\perp W|E$ Not indicated by structure

5. $W \perp\!\!\!\perp N$ True

6. $W \perp\!\!\!\perp E$ Not indicated by structure

7. $W \perp\!\!\!\perp E|K$ Not indicated by structure
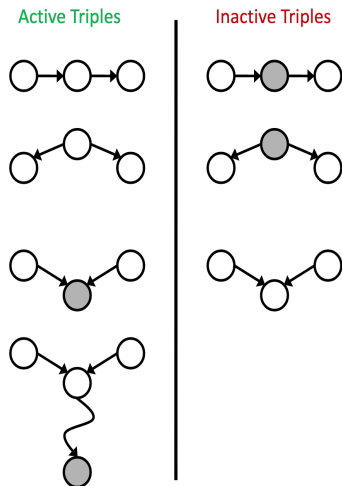
Note that just based on the structure (but without knowing the CPT values), we can assert independence but we cannot assert dependence.